Perceptual Evaluation Methods for Haptics Research
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Basic Concepts, Laws, and Classical Methods in Psychophysics

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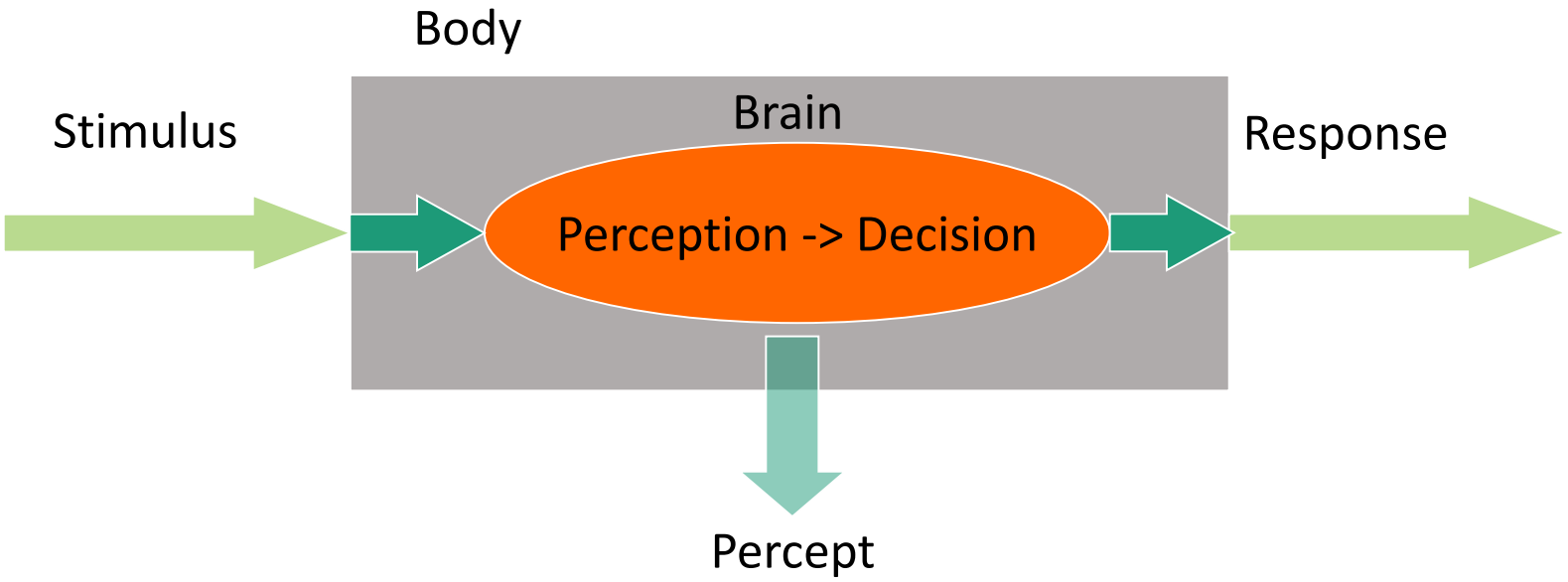
POSTECH

Perception and Psychophysics

- Perception
  - Awareness of the elements of environment through physical sensation (by Merriam-Webster)

- Psychophysics
  - Methodology for investigating relationships between stimuli in the physical domain and sensations in the psychological domain
  - Central to experimental psychology

Essence of Psychophysics



(Stimulus, Response) → (Stimulus, Percept)

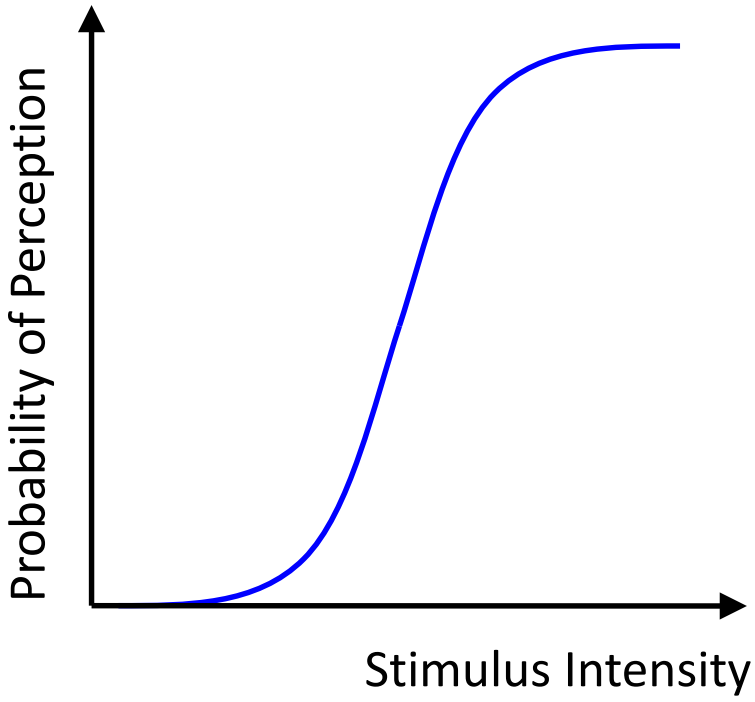
Gustav Fechner (1801 – 1887)
An early pioneer in experimental psychology and founder of psychophysics

Threshold Level vs. Suprathreshold Level

- Increasing the physical energy of stimulus increases the perceived magnitude of sensation.
- Perceptual Threshold
  - The smallest amount of stimulus energy required to produce a sensation
- Perceptual Magnitude
  - The magnitude of a sensation we feel from a physical stimulus
- Two Classes of Psychophysical Methods
  - Threshold Level: stimulus intensity ≈ threshold
  - Suprathreshold Level: stimulus intensity >> threshold

Psychometric Function

- A central concept of psychophysics for the threshold level
- A function from stimulus intensity to the probability of perceiving the stimulus
- Explicitly models the sensory noise of perception process
- Usually a S-shaped ogive
- Modeled as a cumulative normal or logistic distribution

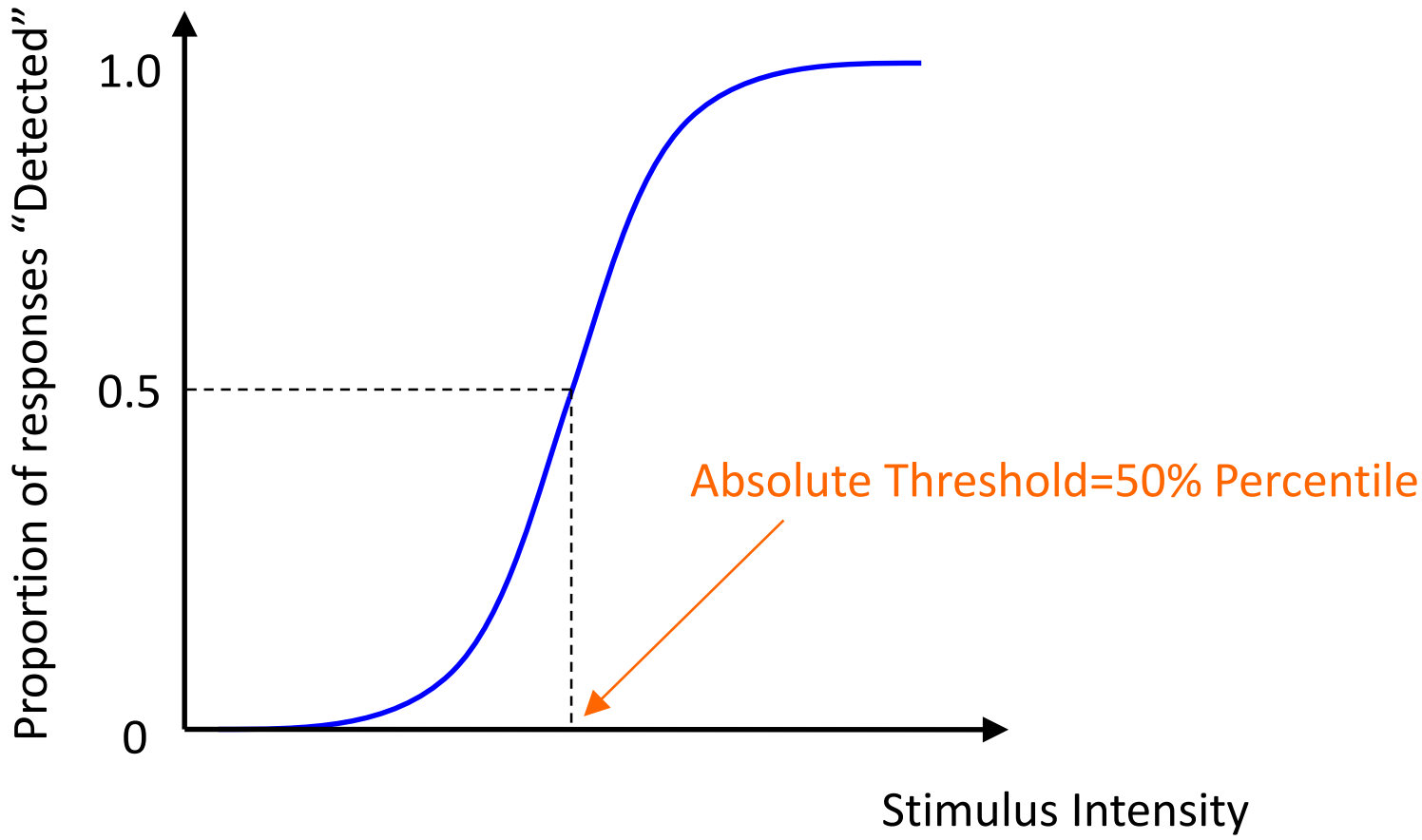


Absolute Threshold

- The smallest amount of stimulus energy to produce a sensation that can be reliably detected.
- Also called detection threshold
- E.g. weight detection of one mass placed on one hand

- 50% percentile is mostly widely used, but other higher percentiles are also frequently used

Absolute Threshold in Psychometric Function

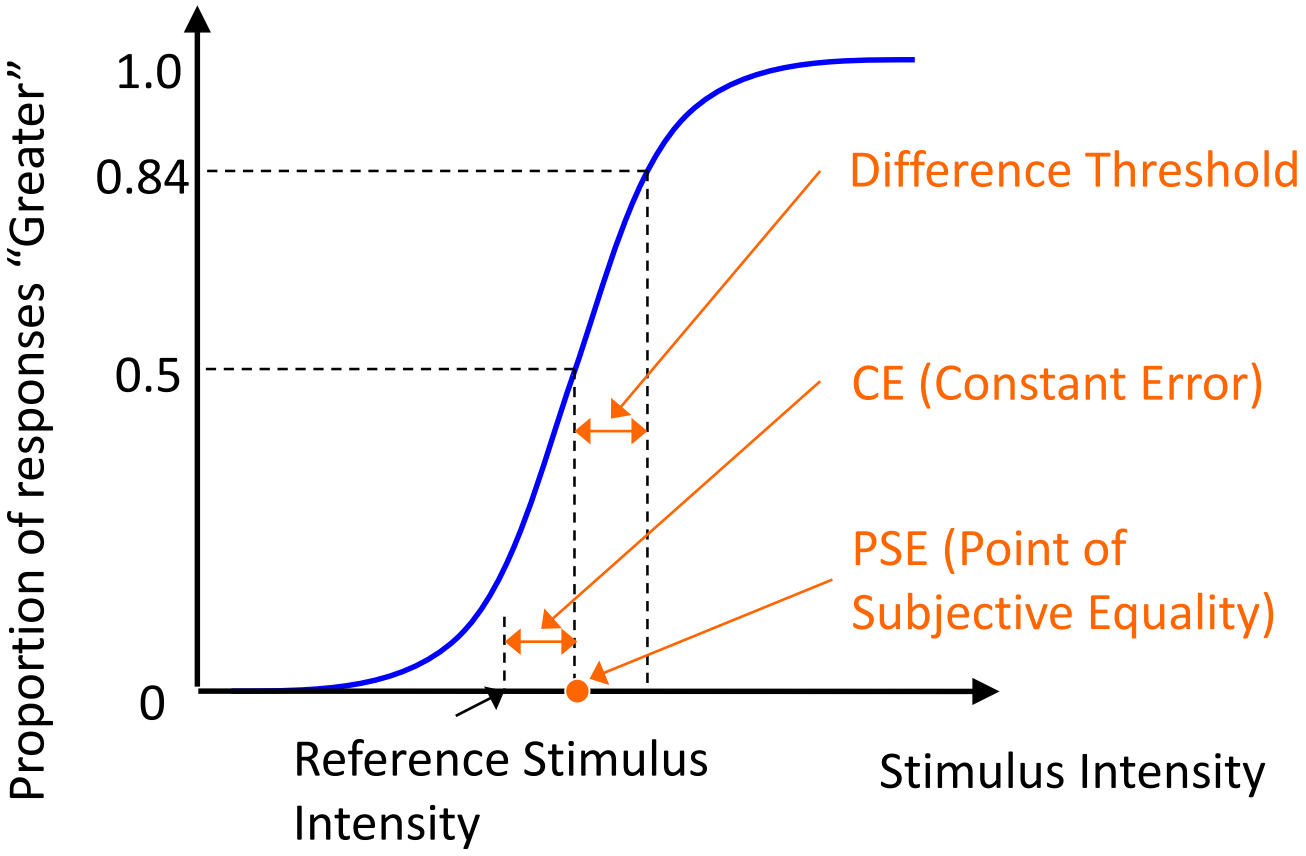


Difference Threshold

- The smallest amount of stimulus energy difference to produce a sensation that can be reliably discriminated
- Also called discrimination threshold
- E.g. weight discrimination of two masses placed on the two hands

- Difference threshold requires the presence of a reference (standard) stimulus

Difference Threshold in Psychometric Function



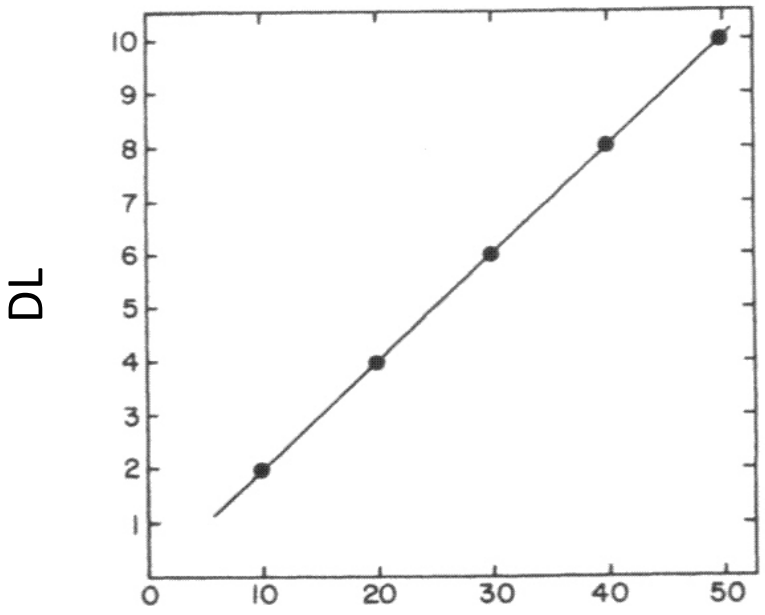
Weber's Law

- Ernst H. Weber (1795 – 1878)
  - A founder of experimental psychology
- Weber's Law
  - Empirical law discovered in weight discrimination
  - Difference threshold is proportional to reference stimulus intensity

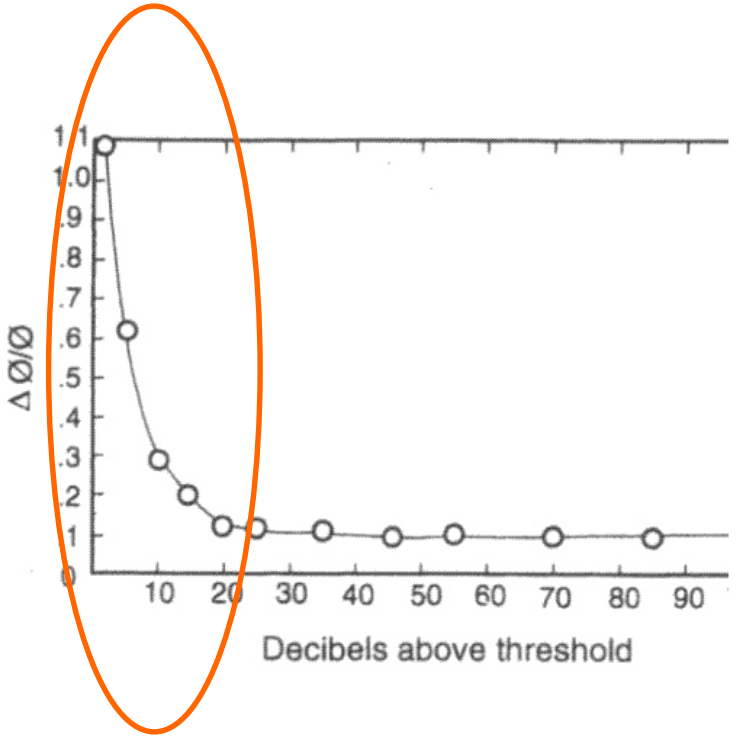
$$DL = w\ I$$

  - DL: Difference Threshold
  - w: Weber Fraction
  - I: Reference Stimulus Intensity
  - Mostly true, but exceptions also exist.

Weber's Law – Graphical Illustration



Reference Stimulus Intensity

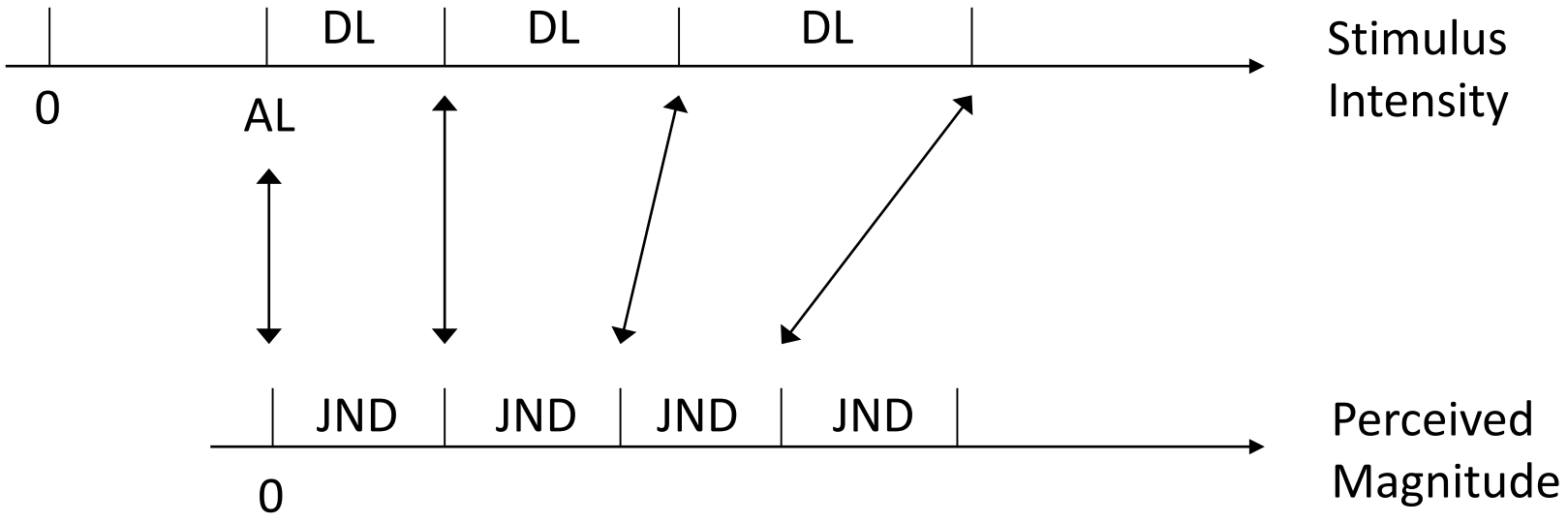


Common when reference stimulus intensities are close to absolute threshold

Perceived Magnitude

- Psychophysical Magnitude Function
  - A mapping from stimulus physical intensity to perceived intensity
- Fechner's Law
  - Proposed by Fechner as a natural extension of Weber's law
  - Applicable only to very limited cases
  - Included for historical reasons
- Steven's Power Law
  - One of the best established empirical laws in psychology
  - Use this form to obtain a psychophysical magnitude function

Fechner's Law – Idea



JND: Just Noticeable Difference (a Unit of Perceived Magnitude)

* Note that DL and JND are used interchangeably in most literature.

Fechner's Law – Equation

- "Elements of Psychophysics," 1860.
- P: Perceived magnitude
  I: Stimulus intensity
  I_0: Absolute threshold
- From Weber's law,

$$dP = c\frac{dI}{I}$$

$$\int_{P_0}^{P} dP = c\int_{I_0}^{I} \frac{dI}{I}$$

$$P - P_0 = c(\log I - \log I_0)$$

$$P_0 = 0$$

Fechner's Law

$$P = c\log\left(\frac{I}{I_0}\right)$$

- Examples where Fechner's law holds well
  - Sound intensity: decibel

Steven's Power Law

- By Stanley S. Stevens (1906-1973)
- Can be viewed as a general form of Fechner's Law

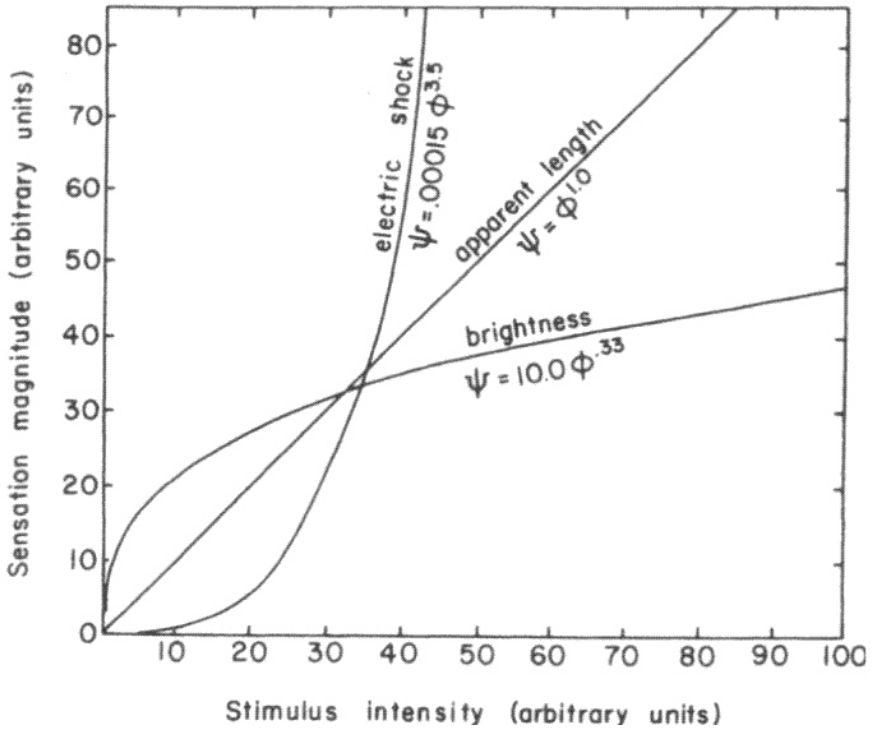
$$P = cI^n \qquad \log P = n \log I + c'$$

$$\text{or} \quad P = c(I - I_0)^n$$

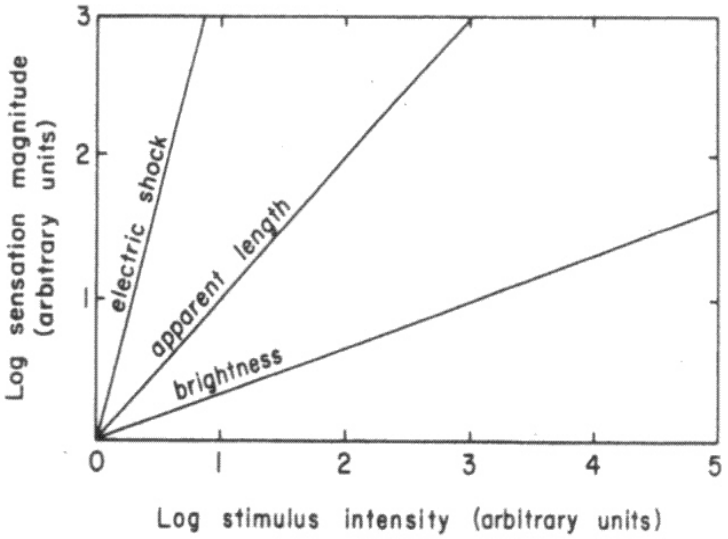
- The exponent *n* depends on stimulus conditions.
  - Brightness: 0.5 (Point source)
  - Vibration: 0.6 (250 Hz on finger)
  - Visual length: 1 (Projected line)
  - Electric shock: 3.5 (Current through fingers)
  - See http://en.wikipedia.org/wiki/Stevens'_power_law for a comprehensive list
- Use the Power law whenever you need to get a magnitude function

Steven's Power Law - Examples

Linear vs. Linear



Log vs. Log



Classical Psychophysical Methods

- Invented by Fechner
- For measuring perceptual thresholds

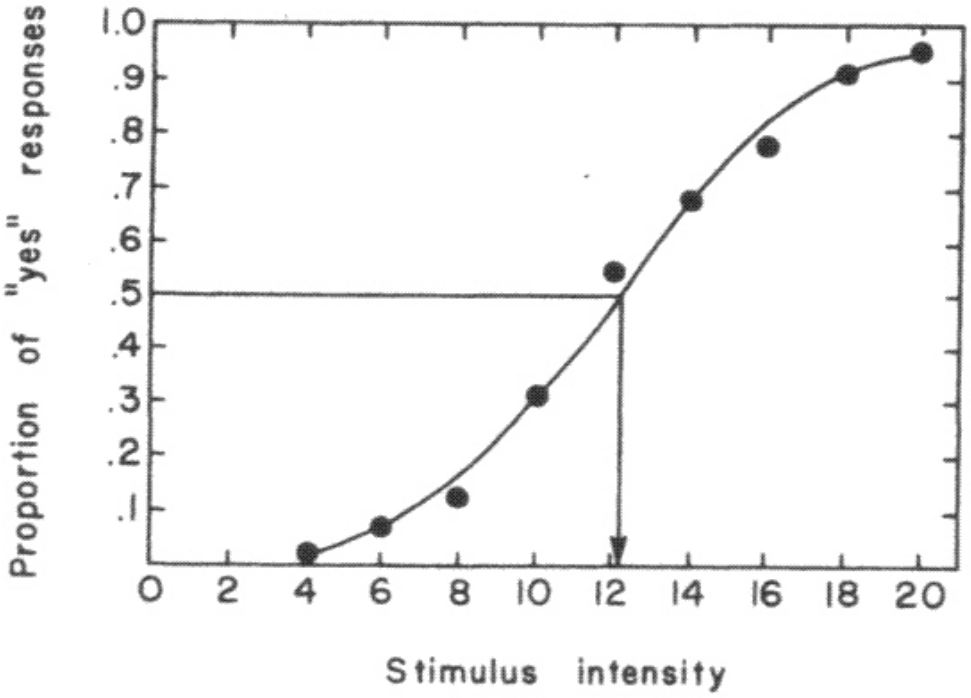
Method	Accuracy	Efficiency	Notes
Method of constant stimuli	Best	Worst	Only method that allows to obtain a whole psychometric function
Method of limits	Middle	Middle	Balanced Can be used for research purposes
Method of adjustment	Worst	Best	Good for pilot experiments or clinical trials Avoid using this for research purposes if possible

Method of Constant Stimuli

- Stimuli
  - A set of stimulus intensities (5-9) that are evenly spaced
- Procedures
  - A stimulus is randomly selected from the stimulus intensity set and presented to the participant
  - The participant is asked to answer
    - Whether the stimulus was detected (for absolute threshold),
    - Whether the test (or comparison) stimulus was greater than the reference stimulus (for difference threshold)
  - Each stimulus intensity should be repeated in a large number of trials (e.g., 100)
  - The time necessary for finish an experiment also needs to be considered to determine the number of trials.

Data Analysis

- Record the proportions of “yes” and plot them against stimulus intensity
- Fit a psychometric function to the recorded data



Psychometric Function Fitting

- Widely-Used Probability Distributions
  - The normal distribution

$$f(x)=\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-\mu)^2}{2\sigma^2}} \qquad F(x)=\frac{1}{2}\left(1+\mathrm{erf}\left(\frac{x-\mu}{\sqrt{2}\sigma}\right)\right), \text{ where } \mathrm{erf}(x)=\frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$$

  - The logistic distribution

$$f(x)=\frac{e^{-\frac{x-\mu}{s}}}{s\left(1+e^{-\frac{x-\mu}{s}}\right)^2} \qquad F(x)=\frac{1}{1+e^{-\frac{x-\mu}{s}}}=\frac{1}{2}\left(1+\tanh\left(\frac{x-\mu}{2s}\right)\right)$$

$$\text{mean}=\mu, \quad \text{s.d}=\frac{\pi}{\sqrt{3}}s$$

- Fit a function of your choice to measured data and determine function parameters (probit analysis for the normal distribution or regular regression analysis)

Finding Thresholds from a Psychophysical Function

- Determine the “level” of correctness in perception
- Detection Experiment
  - 50% detectability (half right and half wrong): AL = mean
  - 84% detectability: AL = mean + standard deviation
  - x% detectability: AL = x % percentile (stimulus intensity that results in the detection probability of x)
- Discrimination Experiment
  - PSE = mean
  - 84% discriminability: DL = standard deviation
  - x % discriminability: DL = x % percentile – mean (PSE)

Methods of Limits – Absolute Thresholds

- The method of limits consists of the same number of ascending and descending series
- Ascending series
  - A series begins with a stimulus intensity well below the threshold.
  - A subject is asked to answer if s/he has detected the stimulus.
  - If the answer is “no”, the stimulus intensity is increased by a small step.
  - Steps 2 and 3 are repeated until the subject changes his or her response from “no” to “yes.”
- Descending series
  - Exactly symmetric

Methods of Limits – Example

for Hearing by the Method of Limits[a]

Stimulus intensity (dB)	*A*	*D*	*A*	*D*	*A*	*D*	*A*	*D*	*A*	*D*
10						Y				
9		Y				Y				Y
8		Y				Y				Y
7		Y		Y		Y				Y
6		Y		Y	Y	Y		Y		Y
5	Y	Y		Y	N	Y	Y	Y		Y
4	N	Y	Y	N	N	N	N	Y	Y	N
3	N	N	N		N		N	Y	N	
2	N		N		N		N	N	N	
1	N		N		N		N		N	
0	N		N				N		N	
−1	N		N				N			
−2	N						N			
−3	N						N			
−4	N									
−5	N									
−6	N									
−7	N									
−8	N									
−9	N									
−10	N									
Transition points =	4.5	3.5	3.5	4.5	5.5	4.5	4.5	2.5	3.5	4.5

[a]Mean threshold value = 4.1

Methods of Limits – Absolute Thresholds (2)

- Cautions
  - The order of series (ascending or descending) should be randomized.
  - Randomize initial stimulus intensity (well above or below an expected threshold). Otherwise, participants tend to rely on memory.
- Data Analysis
  - In each series, a threshold estimate is a mean between two stimulus intensities right before and right after a subject changes his answers from “no” to “yes” (ascending series) or from “yes” to “no” (descending series).
  - The absolute threshold is the mean of the threshold estimates.

Method of Limits - Difference Thresholds

- Ascending series
  - A series begins with a test stimulus with the intensity well below the reference stimulus intensity.
  - A subject is asked to answer if the test stimulus felt “less than”, “equal to”, or “greater than” the reference stimulus.
  - If the answer is “less than” or “equal to”, the stimulus intensity is increased by a small step.
  - Steps 2 and 3 are repeated until the subject changes his response from “equal to ” to “greater than.”

- Descending series
  - Exactly symmetric

Method of Limits - Difference Thresholds (2)

- In each series, two points are recorded.
  - Upper limen (L_u): A point where "greater than" responses changes to an "equal to" response.
  - Lower limen (L_l): A point where "less than" responses changes to an "equal to" response.
- Then,

$$\text{PSE} = \frac{1}{2}\left(\bar{L}_u + \bar{L}_l\right), \text{ and}$$

$$\text{DL} = \frac{1}{2}\left(\bar{L}_u - \bar{L}_l\right),$$

where $\bar{L}_u$ and $\bar{L}_l$ are the means of upper and lower limens, respectively.

DL Measurement – Example

for Hearing by the Method of Limits[a]

Stimulus intensity (dB)	*A*	*D*	*A*	*D*	*A*	*D*	*A*	*D*	*A*	*D*
24.5						G				
24.0		G				G		G		
23.5		G				G		G		G
23.0		G		G	G	G		G		G
22.5		G	G	G	E	G	G	G		G
22.0	G	E	E	G	E	G	E	G	G	E
21.5	E	E	E	E	E	G	E	E	E	E
21.0	E	E	E	E	E	E	E	E	E	E
20.5	E	E	E	E	E	E	E	E	E	E
20.0	E	E	E	E	E	E	E	E	E	E
19.5	E	E	E	E	E	E	E	E	E	E
19.0	E	E	E	E	E	E	E	E	E	E
18.5	E	L	E	E	E	E	E	E	E	E
18.0	E		E	L	E	L	E	L	E	E
17.5	L		L		E		L		L	L
17.0	L		L		L		L		L	
16.5	L		L		L		L		L	
16.0	L				L				L	
15.5	L				L				L	
Upper limen	21.75	22.25	22.25	21.75	22.75	21.25	22.25	21.75	21.75	22.25
Lower limen	17.75	18.75	17.75	18.25	17.25	18.25	17.75	18.25	17.75	17.75

[a]Interval of uncertainty = IU = $\bar{L}_u - \bar{L}_l$ = 22.00 − 17.95 = 4.05. Difference limen = DL = ½ IU = ½ (4.05) = 2.025. Point of subjective equality = PSE = ½($\bar{L}_u + \bar{L}_l$) = ½ (22.00 + 17.95) = 19.97.

Measuring Difference Threshold – Alternative

- There are both ascending and descending series.
- In each trial (either in ascending or descending series), the subject's task is to determine whether the test stimulus is "greater than" or "less than" the reference.
- The stopping rule is the same as that of absolute threshold measurement.
- Then,
  - PSE = the mean of all transition points
  - DL = the standard deviation of all transition points

Method of Adjustment - Procedure

- Absolute Threshold Measurement
  - In each trial, a subject is asked to adjust the stimulus intensity so that it is just barely detectable.
  - AL = the mean of the adjusted intensities.

- Difference Threshold Measurement
  - In each trial, a subject is asked to adjust the test stimulus intensity so that it is perceptually identical to the reference stimulus.
  - PSE = the mean of the adjusted intensities.
  - DL = the standard deviation of the adjusted intensities.

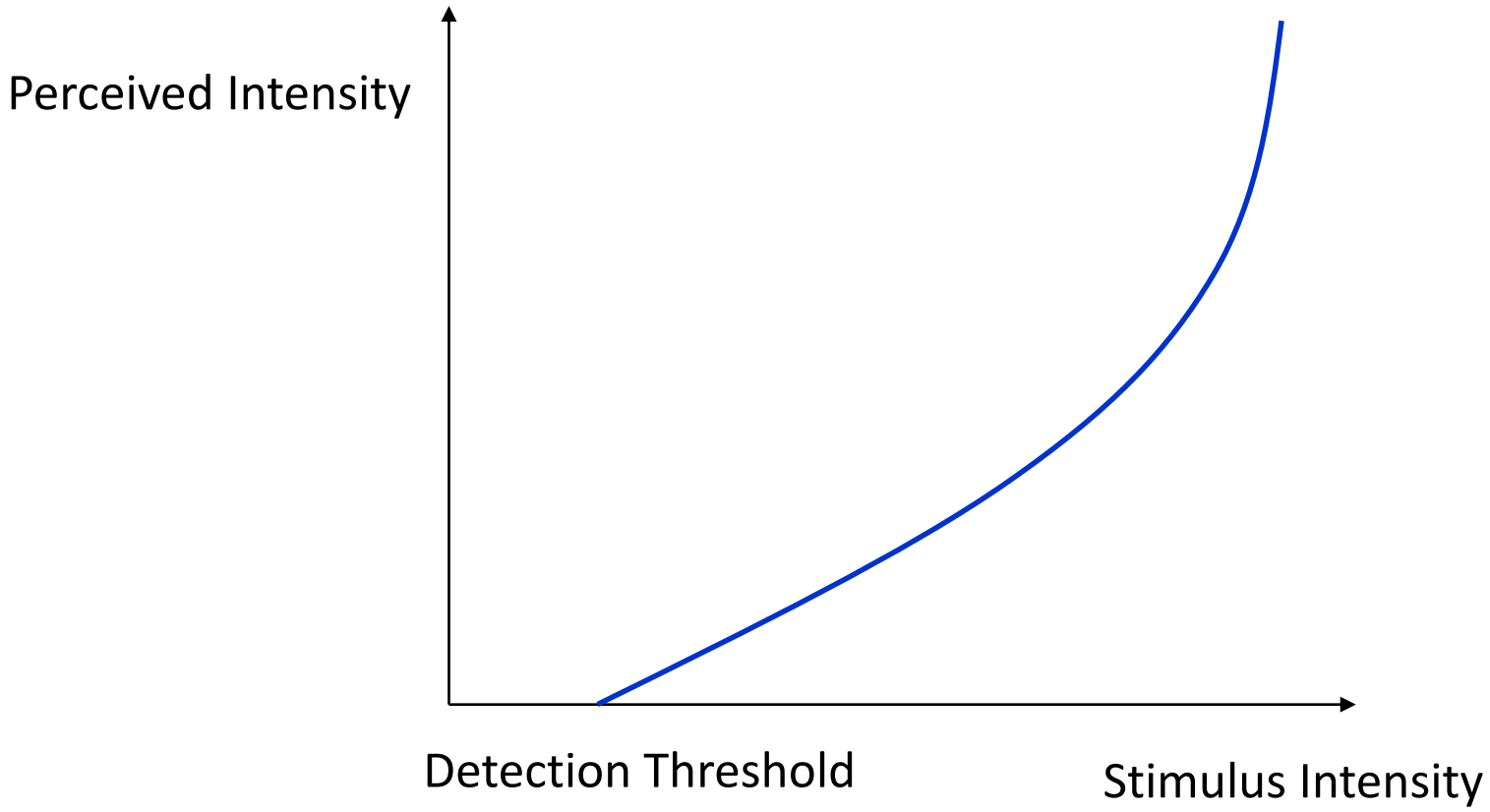
Method of Adjustment – Pros and Cons

- Most efficient
- Generally considered too inaccurate for research purposes due to the excessive control of a subject over stimulus intensities
- Can be applied only when stimulus intensity can be adjusted continuously or in very small steps

Magnitude Estimation

- So far, we have focused on sensory perception in the threshold level.
  - The points where a percept can occur (i.e., detection and difference thresholds) received main attention.
  - The output of the perception process is essentially binary (i.e., can feel it/cannot feel it), although it was formulated in the probabilistic framework.
- We now move on to how to explain the perception process in the supra-threshold level
  - The output of perception process (perceived intensity) varies continuously with respect to the continuous change of input (stimulus intensity).
  - A central concept is a psychophysical magnitude function.

Hypothetical One-Dimensional Psychophysical Magnitude Function



The actual function shape follows Steven's power law.

Psychophysical Ratio Scaling

- Methods for constructing ratio scales of sensations have been extensively studied in the past 35 years.

- Ratio Scaling Methods
  - Ratio Production
  - Ratio Estimation
  - Magnitude Estimation
  - Magnitude Production
  - Absolute Magnitude Estimation

- Magnitude estimation is the most widely used method for psychophysical ratio scaling

Magnitude Estimation with Modulus

- The subject is presented with a standard stimulus, and instructed that the sensation it produces has a certain numerical value (modulus), e.g., 10.
- On subsequent trials, other stimuli are presented, and the subject assigns numbers to their sensations relative to the value of the modulus.
- The participant is instructed to make his judgments reflecting how many times greater one sensation is than another (the ratio between the two sensations).

Magnitude Estimation without Modulus

- No standard stimulus or modulus is defined by the experimenter
- A subject can define his own modulus in the first trial and use it as a modulus in the subsequent trials.
- Other procedures are the same as those of magnitude estimation with modulus.
- Magnitude estimation without modulus is used more frequently.
- In no modulus design, the data of different subjects are combined using the geometric mean.

$$\text{Geometric Mean} = \log^{-1}\left(\frac{\sum \log X}{N}\right)$$

Magnitude Production

- An inverse of magnitude estimation.

- The experimenter tells the numerical value of some sensory magnitude to the subject, and the subject adjusts stimulus intensity to produce the number.
- Magnitude production can be used to confirm the soundness of magnitude estimation.

- Using magnitude estimation and production together can offset any systematic errors inherent in either method.

Absolute Magnitude Estimation

- An absolute scale is a restricted case of ratio scales in which scale values cannot be transformed in any way.
- There has been ample evidence that we use an absolute scale for sensory magnitudes, which may have been fixed at an early age.
- If the relation between a standard stimulus and a modulus arbitrarily assigned by the experimenter is different from what a subject would assign in absolute scaling, the use of the standard stimulus may bias the resulting psychophysical scale.

- At present, the method of absolute magnitude estimation is recommended to prevent the potential biasing effects of a standard stimulus.

Subject Instructions in Absolute Magnitude Estimation

- In this experiment, we would like to find out how intense various stimuli appear to you. For this purpose, I am going to present a series of stimuli to you once at a time. Your task will be to assign a number to every stimulus in such a way that your impression of how large the number is matches your impression of how intense the stimulus is. We all have impressions of how large various numbers are, and impression of how intense various stimuli are. I would like you to assign a number to each stimulus so that your impression of the size of the number matches your impression of the intensity of the stimulus.
- You may use any positive number that appears appropriate to you – whole numbers, decimals, or fractions. Do not worry about running out of numbers – there will always be a smaller number than the smallest you use and a larger on than the largest you use. Do not worry about numbers you assigned to preceding stimuli. Do you have any questions?

Closing Remarks

- There exist many more advanced psychophysical methods, e.g., adaptive procedures for threshold estimation.
- Read relevant books and papers to see the actual designs and parameters of psychophysical experiments.
- Reference
  - G. A. Gescheider, "Psychophysics: The Fundamentals," 3rd ed., Lawrence Erlbaum Associates, Publishers, 1997

Thank you!

- Questions or comments to: choism@postech.ac.kr