Information Theory:

A useful tool to determine channel capacity, the maximum number of stimuli that can be correctly identified, without running multiple experiments

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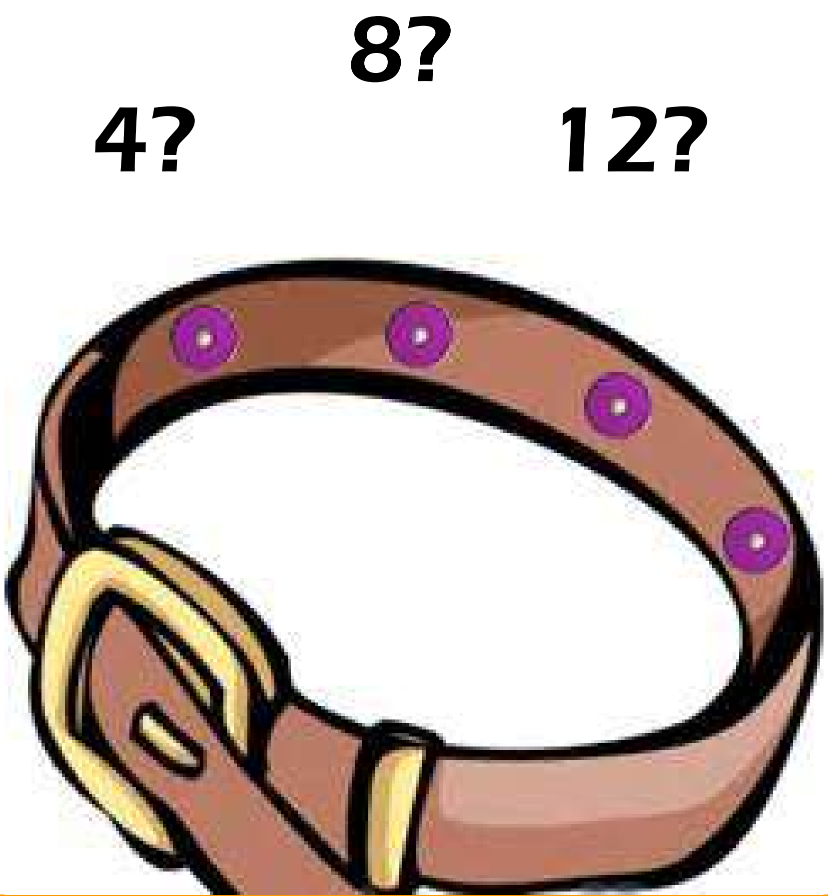
Overview

- Why information theory?
- Information as psychological concepts
- Information transmission measurement
- Channel capacity

WHY INFORMATION THEORY?

How many tactors to put on a belt?



- As number of tactors increases, accuracy (percent-correct scores) decreases
- Objectives:
  - Place maximum number of tactors on the belt, and
  - Achieve a perfect accuracy

➡ Maximum number of perfectly-identifiable tactors

INFORMATION AS PSYCHOLOGICAL CONCEPTS

Information & Uncertainty

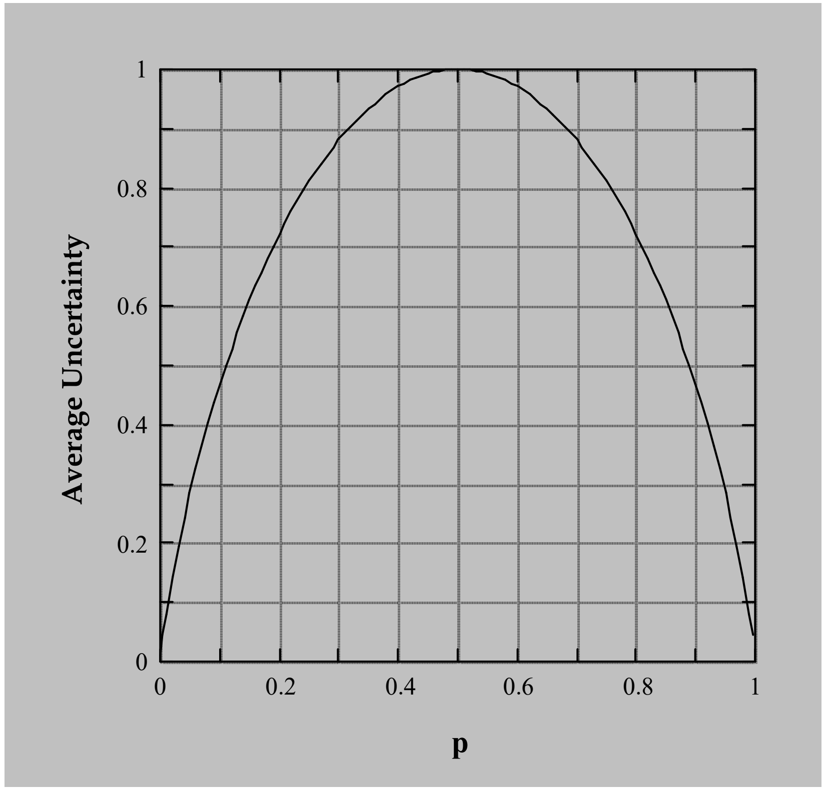
- Information = reduction in uncertainty
- Uncertainty of a given outcome X_i
  - $U_i = \log_2[1/P(X_i)] = -\log_2 P(X_i)$
- If $P(X_i) = 1/k$ (equally likely X_i, $i \in [1, k]$)
  - Then $U_i = \log_2 k$ for all i
- Average uncertainty:
  - $U = \Sigma\, P(X_i)\, U_i$
  - $U = -\Sigma\, P(X_i) \log_2[P(X_i)]$

Average Uncertainty

- Shannon's measure: $U = -\Sigma P(X_i) \log_2[P(X_i)]$
- Unit for uncertainty and information: *bits*

dichotomous distribution with p and q=1–p

average uncertainty:
$-p\log_2 p - q\log_2 q$



INFORMATION TRANSMISSION MEASUREMENT

Absolute Identification Experiment

- One-interval experiment
- Stimuli: S_i, $i \in [1, k]$ $(k > 2)$
- Responses: R_j, $j \in [1, k]$
- One-to-one mapping $(S_i \Leftrightarrow R_i)$
- On each trial, one of the stimuli S_i is presented with an *a priori* probability of $P(S_i)$
- Subject makes a response with R_j
- Trial-by-trial correct-answer feedback is optional

S-R Confusion Matrix (e.g., k = 5)

	R_1	R_2	R_3	R_4	R_5	
S_1	14	3	2	0	1	20
S_2	0	13	2	3	1	19
S_3	4	3	11	1	0	19
S_4	2	0	2	15	1	20
S_5	5	3	2	0	12	22
	25	22	19	19	15	100

# of times S_2 was Presented

# of times the joint event (S_3, R_4) occurred

# of times R_5 was called

IS and IR

- IS (*Information in Stimulus*)
  - IS is the average uncertainty in stimulus

$$IS = -\sum_{i=1}^{k} P(S_i)\log_2 P(S_i)$$

    - ***If all stimuli are equally likely, then***

$$IS = \log_2 k$$

- ***IR (Information in Response)***
  - ***IR is the average uncertainty in response***

$$IR = -\sum_{i=1}^{k} P(R_j)\log_2 P(R_j)$$

IT (Information Transfer)

- Also called "mutual information"
- IT = reduction in uncertainty
- For a particular (S_i, R_j) pair:
  - $U(S_i)$ before: $-\log_2 P(S_i)$
    - Assuming that $P(S_i)$ is constant throughout the experiment
  - $U(S_i)$ after: $-\log_2 P(S_i \mid R_j)$
  - $IT(S_i, R_j) = -\log_2 P(S_i) - [-\log_2 P(S_i \mid R_j)]$

$$\boxed{IT(S_i, R_j) = \log_2 \frac{P(S_i \mid R_j)}{P(S_i)}}$$

- **Average IT = ΣΣ $P(S_i, R_j)\, IT(S_i, R_j)$**
- **IT: the degree of correlation between S's and R's**

DATA ANALYSIS

	R_1	R_2	R_3	R_4	R_5	
S_1	14	3	2	0	1	*20*
S_2	0	13	2	3	1	*19*
S_3	4	3	11	1	0	*19*
S_4	2	0	2	15	1	*20*
S_5	5	3	2	0	12	*22*
	25	*22*	*19*	*19*	*15*	*100*

Estimation of IT — IT_{est}

- Average information transfer:

$$IT = \sum_{j=1}^{k}\sum_{i=1}^{k} P(S_i, R_j)\log_2 \frac{P(S_i|R_j)}{P(S_i)}$$

- ***Its maximum-likelihood estimate:***

$$IT_{est} = \sum_{j=1}^{k}\sum_{i=1}^{k} \left(\frac{n_{ij}}{n}\right)\log_2\left(\frac{n_{ij}\cdot n}{n_i \cdot n_j}\right)$$

where

n_{ij}

$$n_i = \sum_{j=1}^{k} n_{ij} \qquad n_j = \sum_{i=1}^{k} n_{ij}$$

$$n = \sum_{j=1}^{k}\sum_{i=1}^{k} n_{ij} = \sum_{i=1}^{k} n_i = \sum_{j=1}^{k} n_j$$

- ***Interpretation of 2^{IT} or $2^{IT_{est}}$ (compare with $k=2^U$)***

Percent-correct scores and IT_{est}

$$IT_{est} = \sum_{j=1}^{k}\sum_{i=1}^{k}\left(\frac{n_{ij}}{n}\right)\log_2\left(\frac{n_{ij}\cdot n}{n_i \cdot n_j}\right)$$

(A)

25	25
25	25

50%
0 bits

(B)

25	25	25	25
25	25	25	25
25	25	25	25
25	25	25	25

25%
0 bits

(C)

25	0	0	0
0	25	0	0
0	0	25	0
0	0	0	25

100%
2 bits

(D)

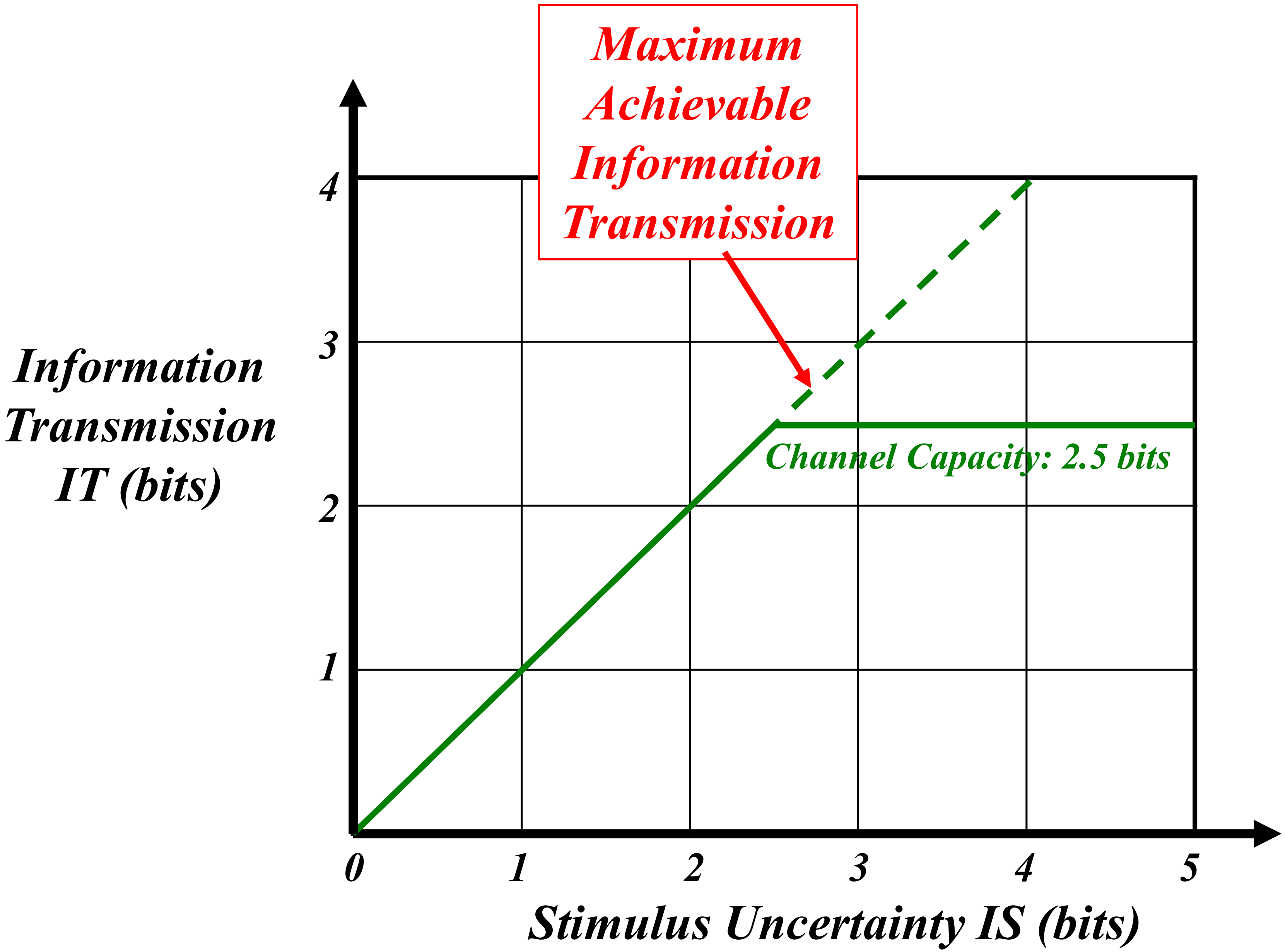
0	0	0	25
0	0	25	0
0	25	0	0
25	0	0	0

0%
2 bits

CHANNEL CAPACITY

Maximum Information Transmission

- Mathematically, $IT \leq IS$.
- Intuitively, if the input and output are perfectly correlated, then $IT = IS$ (= IR).
- Assume that there exists a *maximum* information transmission
  - For small values of IS, $IT = IS$.
  - As IS increases, IT = *constant* regardless of the value of IS.
- This maximum IT is accepted as the channel capacity


Maximum Achievable Information Transmission
Information Transmission IT (bits)
4
3
2
1
0
1
2
3
4
5
Channel Capacity: 2.5 bits
Stimulus Uncertainty IS (bits)

The “Magic Number” 7±2

- The “magic number” is derived from an IT range of 2.3 – 3.2 *bits*
- The “magic number” summarizes the typical channel capacity for uni-dimensional stimuli
- Uni-dimensional stimuli
  - Only one physical variables (target) is manipulated to form the stimulus set
  - Other physical variables (background) are either held constant or randomized

G. A. Miller, “The magical number seven, plus or minus two: Some limits on our capacity for processing information,” The Psychological Review, vol. 63, pp. 81-97, 1956.

EXAMPLES OF HAPTIC CHANNEL CAPACITY

Finger-span Length Identification

Durlach, Delhorne, Wong, Ko, Rabinowitz, & Hollerbach (1989). Manual discrimination and identification of length by the finger-span method. *Perception & Psychophysics*, 46(1), 29–38.

Length Identification Results

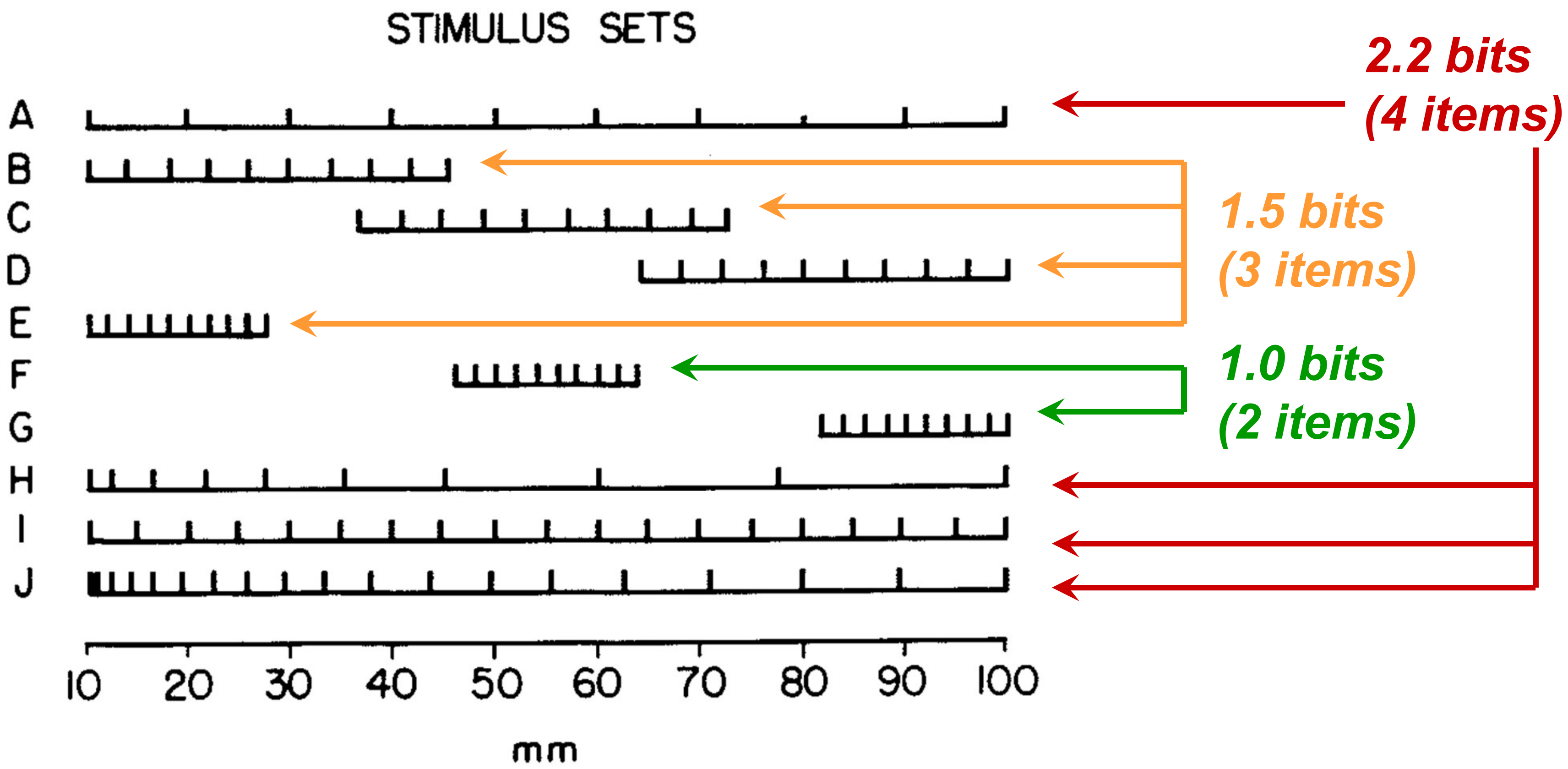
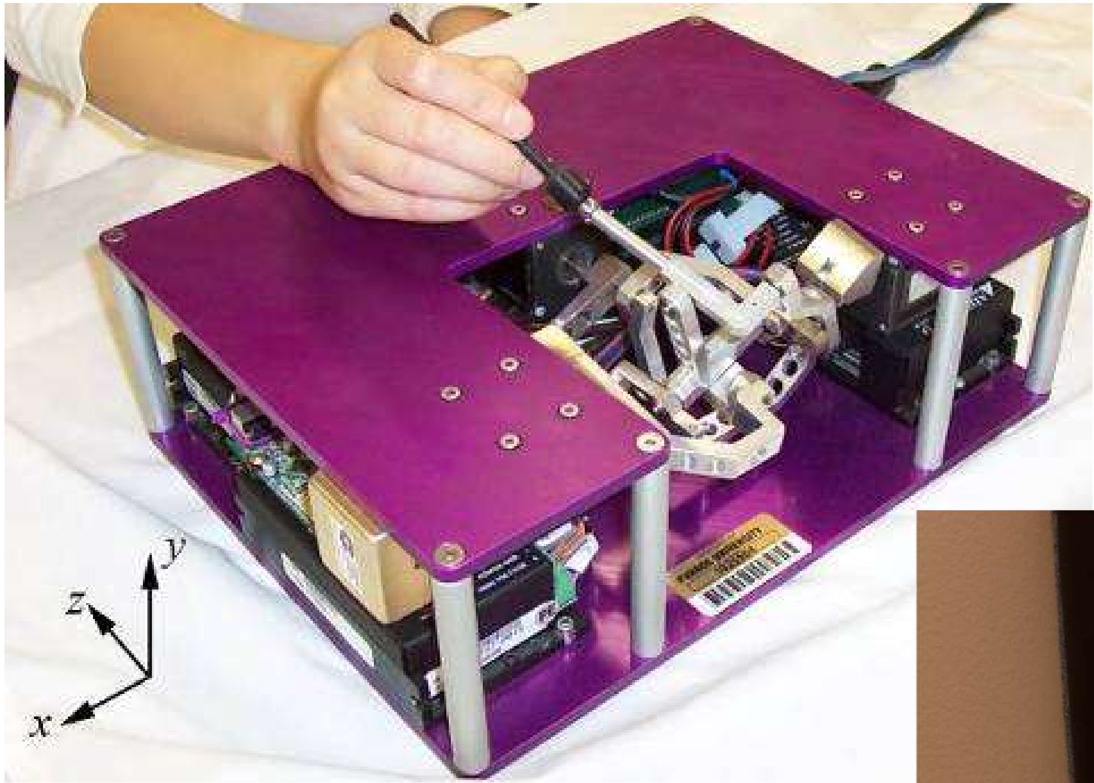
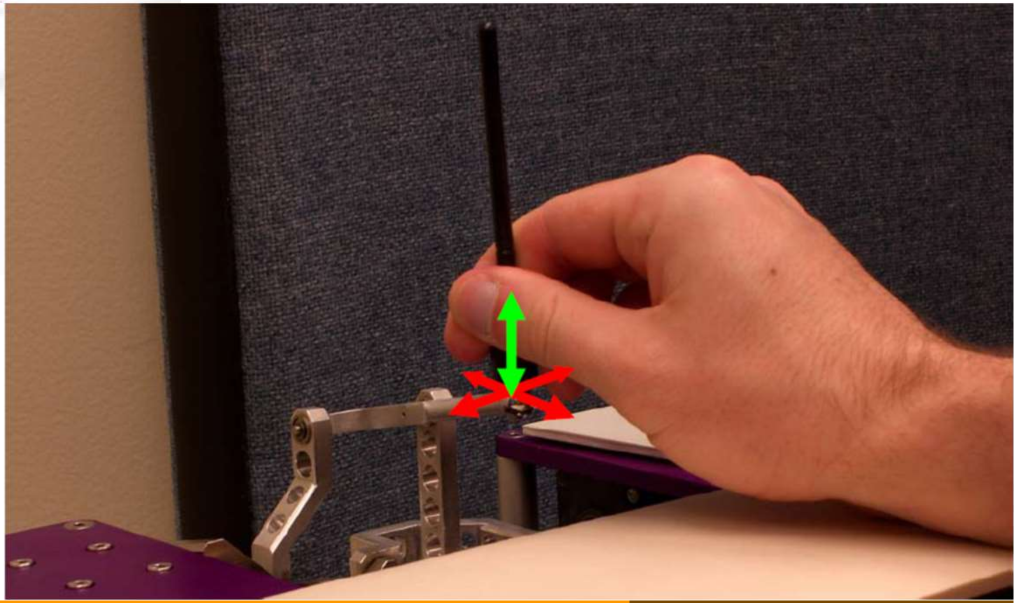


Figure 6. Stimulus sets used in the identification experiments (Experiments 2A and 2B). Sets A–G and I are linear; Sets H and J are approximately logarithmic. Each of Sets A–H have 10 elements; each of Sets I and J have 19 elements.

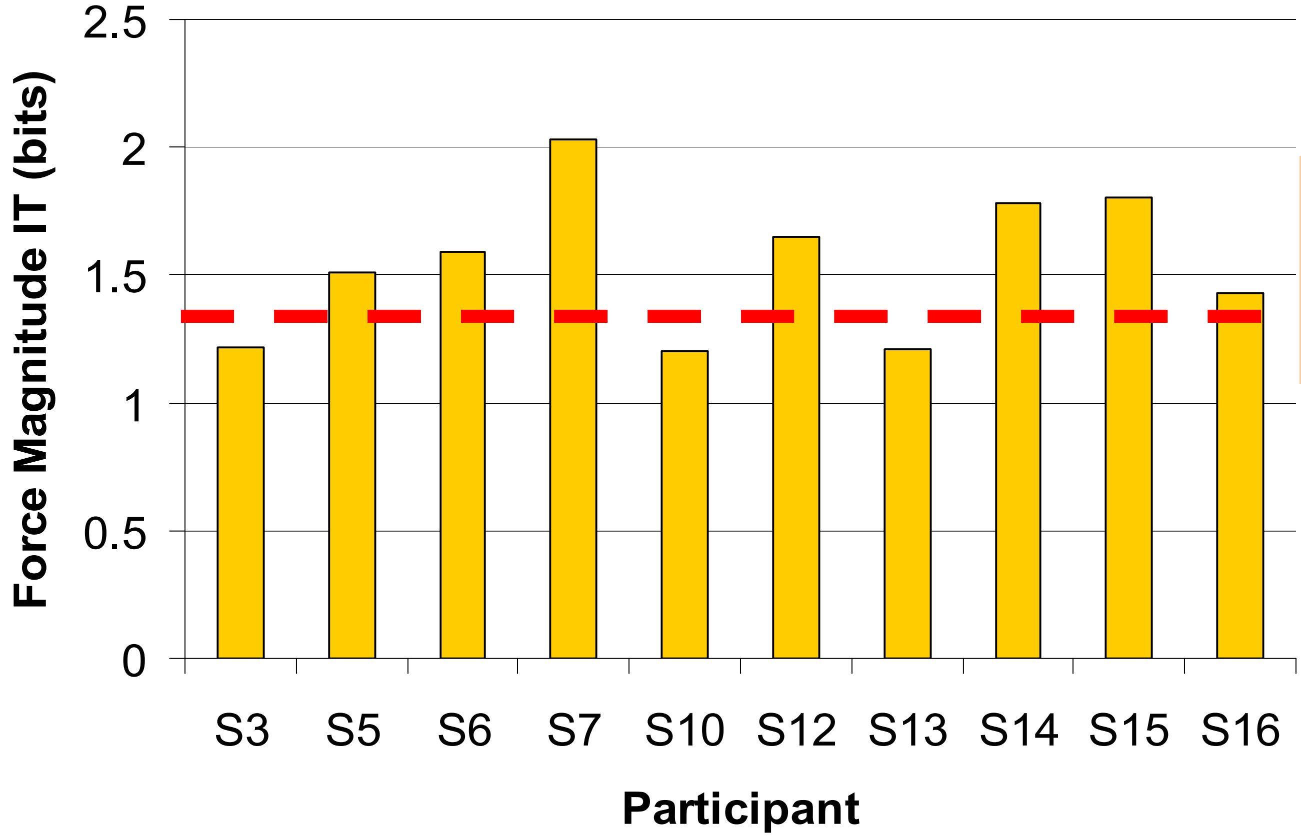
Identification of Force/Stiffness: Towards Haptic "Glyphs"





Force Identification Results

Force range: 0.1 - 5 N



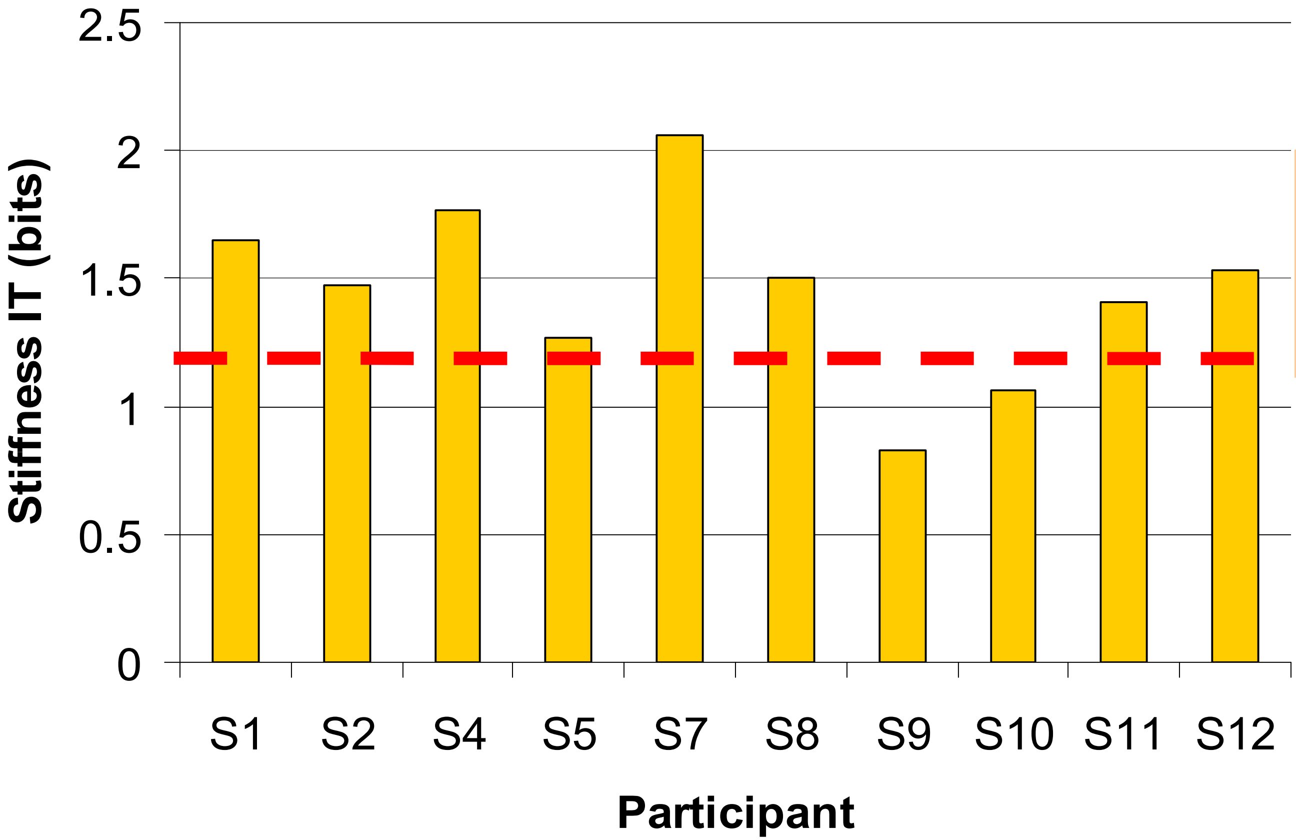
2^{IT} =

2.9 items

Cholewiak, Tan, and Ebert (2008). "Haptic identification of stiffness and force magnitude," *Proceedings of the Symposium on Haptic Interfaces for Virtual Environment and Teleoperator Systems*, pp. 87-91.

Stiffness Identification Results

Stiffness range: 0.2 - 3 N/mm



2^{IT} =

2.8 items

Cholewiak, Tan, and Ebert (2008). "Haptic identification of stiffness and force magnitude," *Proceedings of the Symposium on Haptic Interfaces for Virtual Environment and Teleoperator Systems*, pp. 87-91.

Summary

For tasks involving the identification of one of many stimulus alternatives,

- ❖ Conduct an absolute identification experiment using a number of alternatives (k) that is expected to lead to errors (i.e., beyond channel capacity)
- ❖ Compute the stimulus-response confusion matrix for each participant
- ❖ Average estimated IT values across participants
- ❖ Compute $2^{IT_{est}}$, the total number of stimuli that can be perfectly identified; i.e., channel capacity

Then design your interface accordingly. There is no need to conduct more than one experiment.

Reference

Jones, L. A., & Tan, H. Z. (2013). Application of psychophysical techniques to haptic research. *IEEE Transactions on Haptics*, 6(3), 268-284.

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