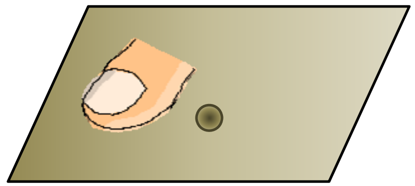
How to organize the perceptual space of multiple stimuli

Shogo Okamoto
Nagoya University

How stimulus-detection process is modeled?

- Stimulus-detection task
  - Judge the presence of a stimulus



Detection of a small bump
on a flat surface

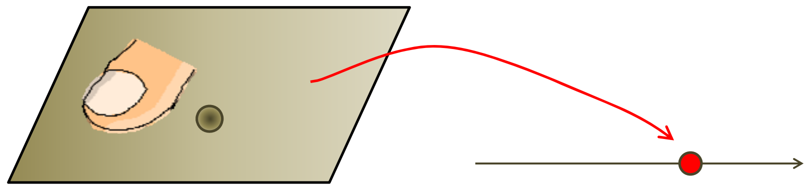
How stimulus-detection process is modeled?
Assumption

- Stimulus-detection task
  - Judge the presence of a stimulus

- Assumption
  - Humans judge a single attribute of stimulus (single dimension)
  - Repeated presentations generate a distribution of values on the single continuum

How stimulus-detection process is modeled?
Continuum

- Distribution on the continuum



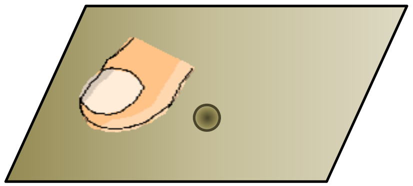
Detection of a small bump
on a flat surface

Internal response caused
by a stimulus

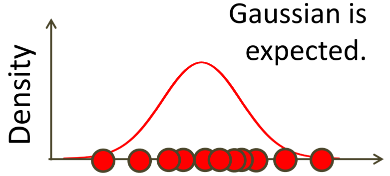
One scan produces one continuous value or
psychophysical response.

How stimulus-detection process is modeled?
Continuum

- Distribution on the continuum



Detection of a small bump on a flat surface

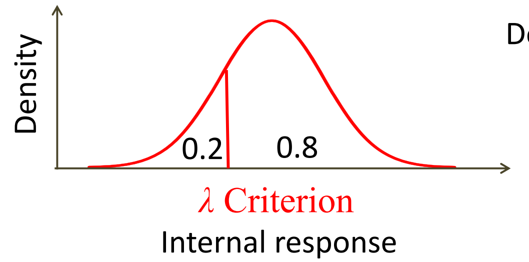


Internal response caused by a stimulus

Multiple presentations of the same stimulus produce a distribution of responses.

How stimulus-detection process is modeled?
Criterion

- A criterion for an optimal explanation of human responses



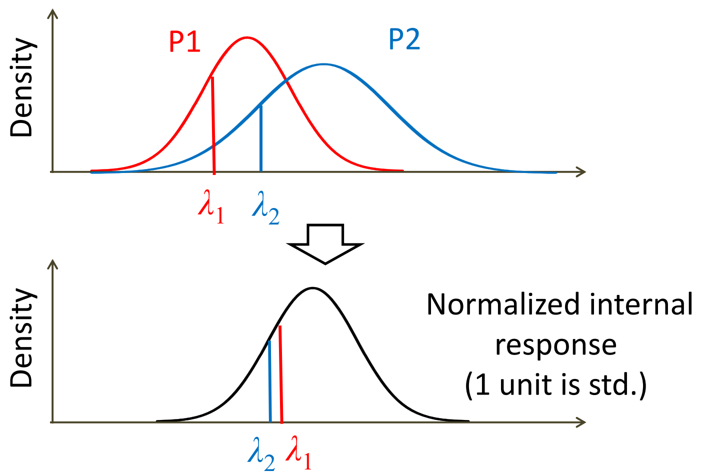
Detection rates

Hit = 0.8
Miss = 0.2

- The hit rate is the proportion of area above λ.
- Misses are the proportion of left area.

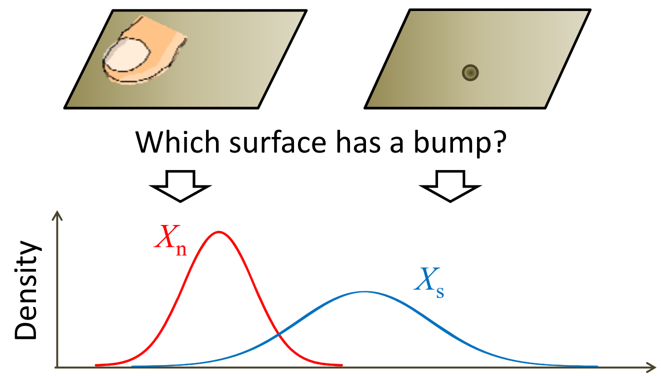
How stimulus-detection process is modeled?
Normalized response

- Different participants produce different distributions.



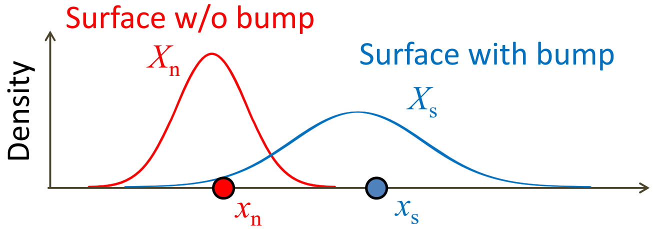
How is the forced choice process modeled?

- 2-alternative forced choice task



- Two types of stimuli lead to two independent distributions. (Assumption)
- Even the surface without a bump produces internal responses.

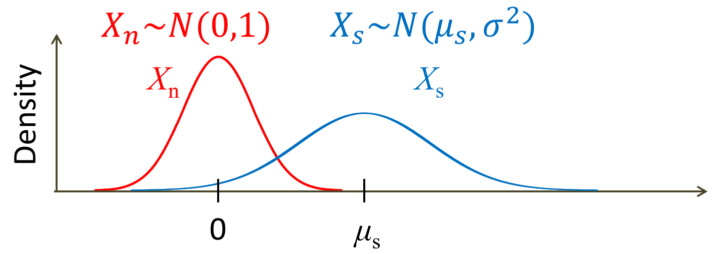
How is the forced choice process modeled?
Rate of correct response



If $x_s > x_n$, then the bumped surface is correctly selected.

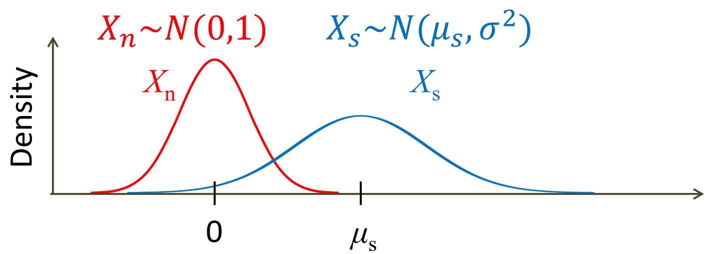
$$\text{Rate of correct response} = \frac{\text{Number of correct responses}}{\text{Number of trials}}$$
$$= P(X_s > X_n)$$
$$= P(X_s - X_n > 0)$$

How is the forced choice process modeled?
Combined Gaussian



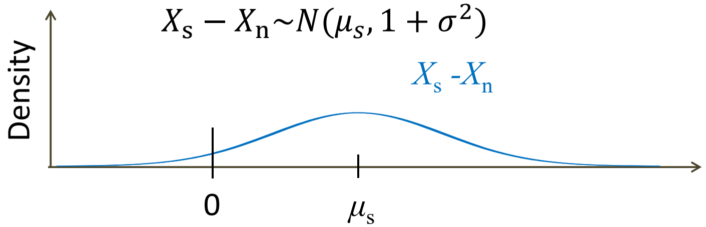
Rate of correct response $= P(X_s - X_n > 0)$

How is the forced choice process modeled?
Combined Gaussian

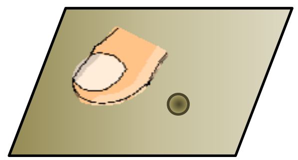


Rate of correct response $= P(X_s - X_n > 0)$

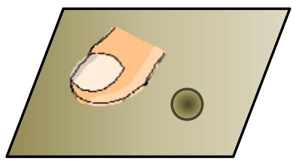
$X_s - X_n \sim N(\mu_s, 1 + \sigma^2)$



How is the identification process modeled?



Small bump



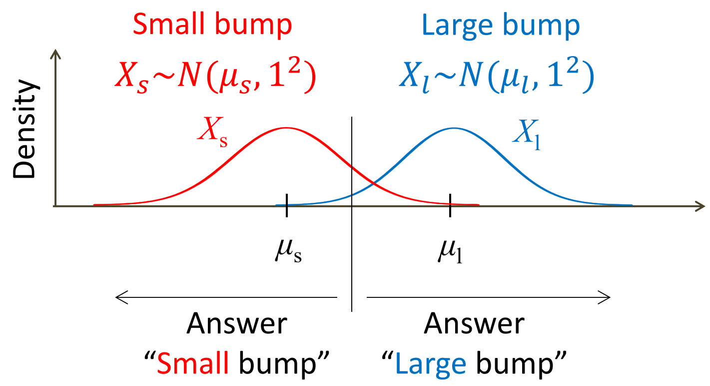
Large bump

Identification of two types of stimuli. Experience either of the small or large bumps, and discriminate them.

Stimuli \ Response	Small	Large
Small	P_{ss} (Correct)	P_{sl}
Large	P_{ls}	P_{ll} (Correct)

Answer ratio table (Confusion matrix)

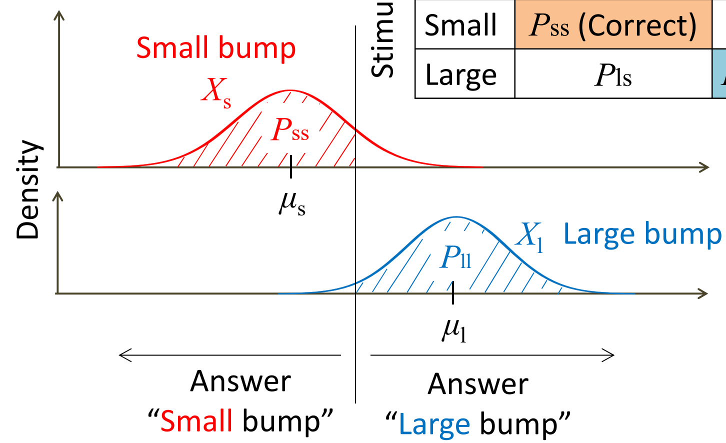
How is the identification process modeled?



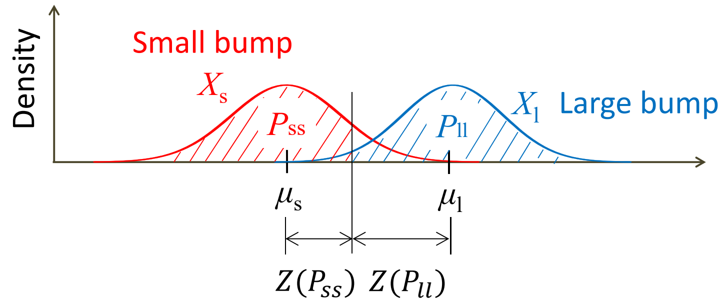
How is the identification process modeled?

Response

Stimuli	Small	Large
Small	P_{ss} (Correct)	P_{sl}
Large	P_{ls}	P_{ll} (Correct)



How is the identification process modeled?
Distance between the stimuli



$$Discriminability = \mu_1 - \mu_2 = Z(P_{ss}) + Z(P_{ll})$$

An index to indicate how easy or difficult to discriminate two stimuli, being independent from the criterion.

How is the identification process modeled?
Distance between the stimuli: Examples

Response

Stimuli	Small	Large
Small	0.7	0.3
Large	0.2	0.8

$$Discrim. = Z(0.7) + Z(0.8) \\ = 0.52 + 0.84 \\ = 1.36$$

Stimuli	Small	Large
Small	0.61	0.39
Large	0.14	0.86

$$Discrim. = Z(0.61) + Z(0.86) \\ = 0.28 + 1.08 \\ = 1.36$$

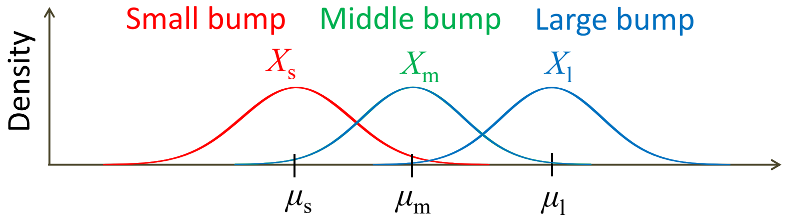
Stimuli	Small	Large
Small	0.9	0.1
Large	0.47	0.53

$$Discrim. = Z(0.9) + Z(0.53) \\ = 1.28 + 0.08 \\ = 1.36$$

Identification of 3 stimuli

Response

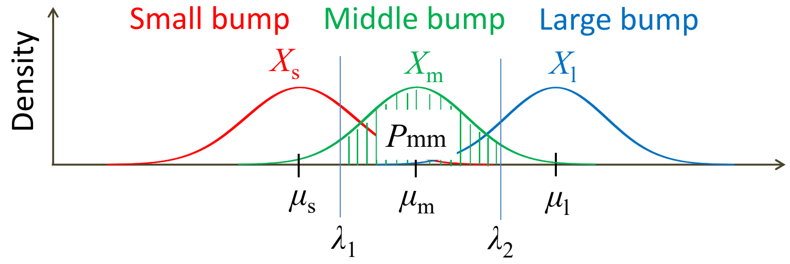
Stimuli	Small	Middle	Large
Small	P_{ss}	P_{sm}	P_{sl}
Middle	P_{ms}	P_{mm}	P_{ml}
Large	P_{ls}	P_{lm}	P_{ll}



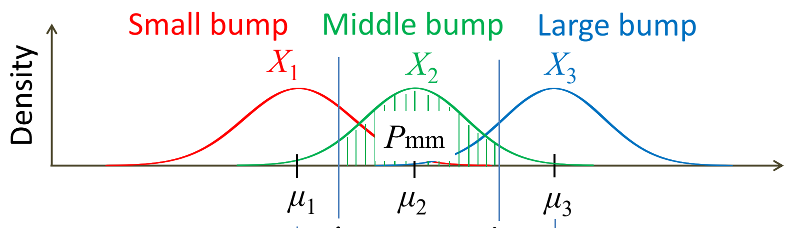
Identification of 3 stimuli

Response

Stimuli	Small	Middle	Large
Small	P_{ss}	P_{sm}	P_{sl}
Middle	P_{ms}	P_{mm}	P_{ml}
Large	P_{ls}	P_{lm}	P_{ll}



Identification of 3 stimuli
Calculation of distance



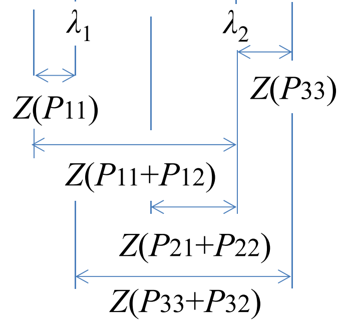
λ_1-μ_1=$Z(P_{11})$

λ_2-μ_1=$Z(P_{11}+P_{12})$

λ_2-μ_2=$Z(P_{21}+P_{22})$

μ_3-λ_1=$Z(P_{33}+P_{32})$

μ_3-λ_2=$Z(P_{33})$



Identification of 3 stimuli
Calculation of distance

Response

Stimuli	Small	Midd	Large
Small	0.7	0.25	0.05
Midd	0.2	0.5	0.3
Large	0.02	0.18	0.8

λ_1-μ_1=$Z(P_{11})$ = 0.52

λ_2-μ_1=$Z(P_{11}+P_{12})$ = 1.65

λ_2-μ_2=$Z(P_{21}+P_{22})$ = 0.52

μ_3-λ_1=$Z(P_{33}+P_{32})$ = 2.05

μ_3-λ_2=$Z(P_{33})$ = 0.84

Identification of 3 stimuli
Calculation of distance

Response (columns) / Stimuli (rows)

	Small	Midd	Large
Small	0.7	0.25	0.05
Midd	0.2	0.5	0.3
Large	0.02	0.18	0.8

$\lambda_1\text{-}\mu_1 = Z(P_{11}) = 0.52$

$\lambda_2\text{-}\mu_1 = Z(P_{11}+P_{12}) = 1.65$

$\lambda_2\text{-}\mu_2 = Z(P_{21}+P_{22}) = 0.52$

$\mu_3\text{-}\lambda_1 = Z(P_{33}+P_{32}) = 2.05$

$\mu_3\text{-}\lambda_2 = Z(P_{33}) = 0.84$

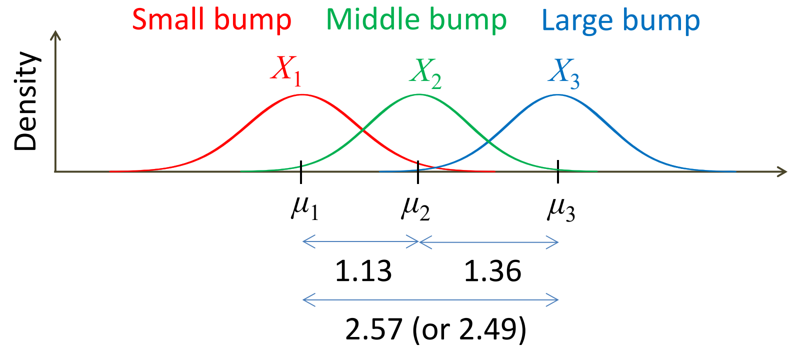
$\boxed{\mu_3\text{-}\mu_1 = 2.57}$

$\boxed{\mu_2\text{-}\mu_1 = 1.13}$

$\boxed{\mu_3\text{-}\mu_1 = 2.49}$

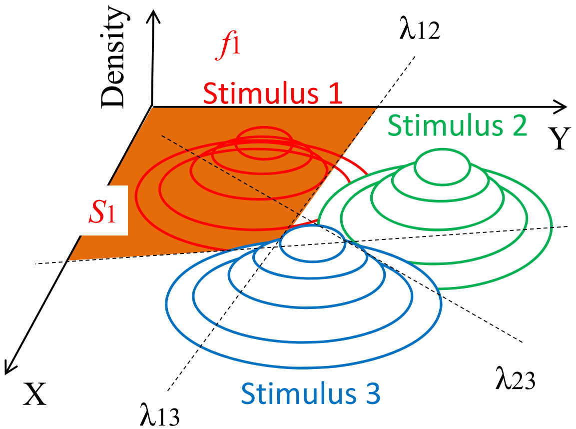
$\boxed{\mu_3\text{-}\mu_2 = 1.36}$

Identification of 3 stimuli
Calculation of distance



Solution of the distances are given by over-determined problem, which is resolved by a least-squared method.

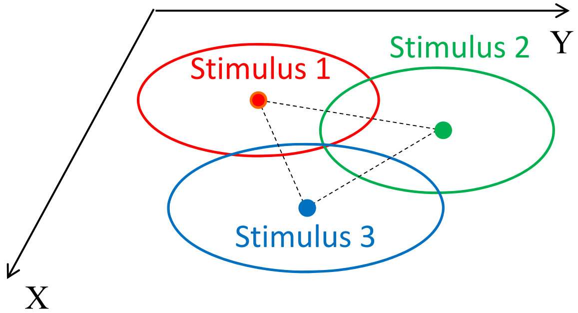
How to model identification problem on a multidimensional space?



- Probability density functions are multidimensional
- Criteria are defined by lines or planes
- Probabilities are computed by area integral

$$P_{11} = \oiint f_1(x, y)\, dS_1$$

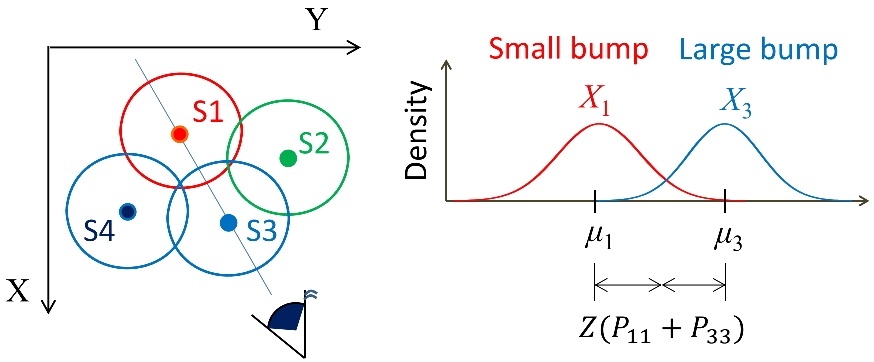
Identification of multidimensional stimuli?
Distance between distribution centers



- Distances between the stimuli are important (rather than the criterion).

- The problem: How to compute the inter-stimulus distances from the confusion matrix.
  - Possible to rely on a nonlinear optimization engine e.g., fmin of Matlab.

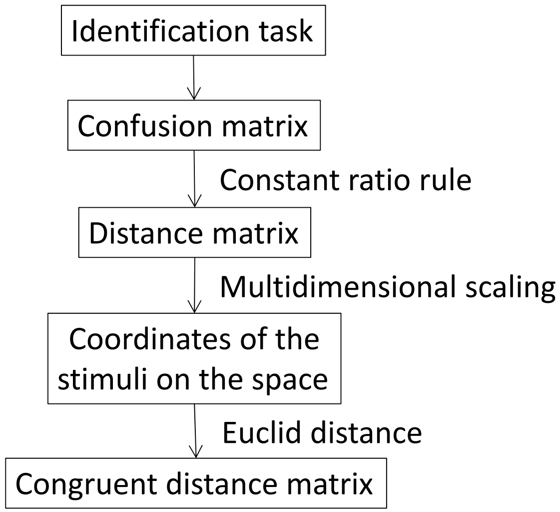
Identification of multidimensional stimuli?
Constant ratio rule



Constant ratio rule

The distance between two stimuli on the multidimensional perceptual space is well approximated by a distance calculated by only those two stimuli.

Identification of multidimensional stimuli?
Congruent distance matrix



What we have glanced at today

- Signal detection theory for multiple stimuli
  - How the stimulus detection process is modeled
  - How the 2-alternative-forced-choice process is modeled
  - How the identification of two stimuli is modeled
  - How the identification of more-than-three stimuli is modeled
- There are several methods to interpret the confusion matrices, however, the one based on the signal detection theory is …